Optimal Servo Design for distributed parameter system defined by a bilinear partial differential equation

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Abstract

We propose a mathematical model described in a bilinear partial differential equation of nonlinear coupled with state variable and control variable in this study. Furthermore, we propose the optimal servo system design of the stochastic model considering the disturbance. The general mixing reaction system is described in a bilinear partial differential equation.

The complete mixing reaction system is assumed to maintain a space uniformly, we construct a bilinear model lumped parameter system. The target model is linearized by using a exact linear theory, we present a design proposal on the optimal servo system considering a disturbance into the system.

Keyword: bilinear partial differential equation, exact linear theory, observer theory, optimal control.

1   Introduction

Current computer technology has resulted in contributed greatly to the field of science and technology or engineering. The computer also has enabled high complex calculations and precision system control. With respect to the field of engineering, there is remarkable in the field of system control. One of the important factors that makes possible to apply modern control theory is to construct a mathematical model.

With respect to mathematical modeling, we have reported that by creating a state in which the production density of each process corresponds to physical propagation, the manufacturing process is most appropriately described using a diffusion equation [1]. In other words, if the potential of the production field (stochastic field) is minimized, the equation is defined by the production density function $S_i(t,x)$ and the constraint is described using an advective diffusion equation to determine the transportation speed ρ [1]. On the other hand, with respect to a bilinear partial differential equation (BPDE), Dr.Shima proposed an optimal control system for heat exchanging system. The mathematical model is described by BPDE [2].

In our previous study, we proposed a mathematical model for a thermal reaction process of external heating equipment. The new control system design for this process, which treats a heat source flowing model for an externally attached device is proposed. The equation of a distributed parameter system as a coupled system with the heat reaction process is presented [3]. We also proposed that the target control system can be configured using the control parameter of the overall heat exchange coefficient (OHEC), which is given using a linear approximation from BPDE to an ordinary differential equation (ODE). Generally, in case of a physical variable in the mathematical model, there are two forms that depend on only a time and a time/space . In only time dependency, the mathematical model is described by an ordinary differential equation. In the time/space dependency, it is described by a partial differential equation. Furthermore, it is sometimes described by a stochastic differential equation by considering a disturbance (noise) to the system from outside. We have reported on a production process represented by a partial differential equation (including stochastic systems) in our previous research[1, 3, 4, 5].

With respect to the stochastic modeling of our previous report, a given control equipment is ordered from a customer, then manufactured in a manner classified into a number of production elements, and a finished product is delivered to the customer. The feature is in a point that production elements in manufacturing processes are treated as stochastic production operation. In particular, in order to analyze a manufacturing process as a stochastic process, we have introduced an idea of a production level corresponding to an energy level being discussed in physics. A valence electron transits to a conducting state due to a rise in potential (transition of a manufacturing process), and lowers an energy level by radiating energy with time[6].

In this study, we propose a bilinear system model coupled of the state variable nonlinearly and the control variable (external force function), and also propose an optimal servo design, which is derived by a bilinear system model by considering stochastic disturbance. We target a batch mixed reaction model often used in

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chemical processes as a specific system model. Generally, the state variable of such system is described by a partial differential equation as a time and space variables. Therefore, we describe a model of a general mixed reaction system (hereinafter referred to as a distributed parameter system) and present a simple model of a bilinear system. Next, assuming complete uniform mixing of space, we describe a bilinear model in a lumped parameter system and converting it to a linear model by using an exact linearization theory[7, 8]. As a result, we propose an optimal servo system design considering a disturbance as a mathematical approach.
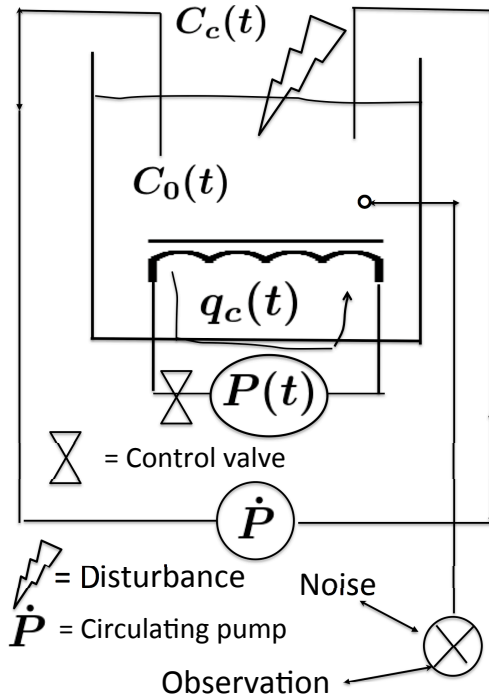
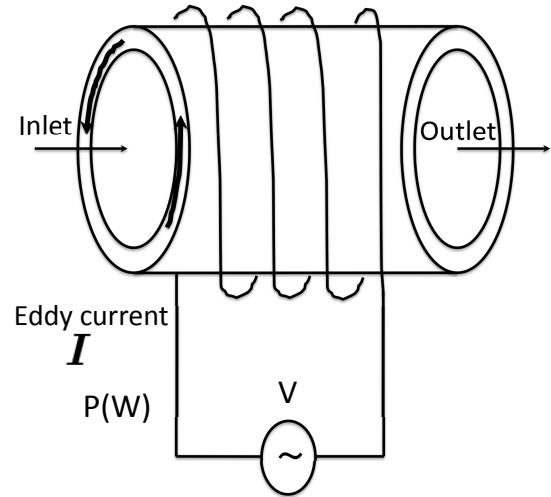Fig. 1: Completely mixed and stirred tank

Fig. 2: Schematic of the induction heating

2 System description of a fully mixed and stirred tank

Figures.1 and 2 show the fully mixed and stirred tank, which is mixed and stirred by a stirrer. Further, the stirring material (liquid) is circulated in the circulation section. The control section is heated by an external medium and has the control function W (t). Table 1 shows the symbols used in Figure 1.

Table. 1: Physical meaning of each symbol

$C_0(t)$	Heated temperature
$C_c(t)$	Ambient temperature
$1/R$	Heat dissipation resistance
C_M	Liquid heat capacity
U	Overall transfer function
A	Cross-sectional area of the heating unit
$P(t)$	Supply potential of the heating unit
$q_c(t)$	Fluid flow rate of the heating unit

We represent the mathematical model for the fluctuation of the heated temperature as follows[3]:

$$\Delta C_0(t) = -\frac{1}{RC_M}\Big\{C_0(t) - C_c(t)\Big\}\Delta t + \frac{1}{C_M}U \cdot A\Big\{q_c(t) - C_0(t)\Big\}\Delta t \tag{2.1}$$

From Equation(2.1), we obtain as follows[3]:

$$\frac{d\Theta_0(t)}{dt} = -\frac{1}{RC_M}\Theta_0(t) + \frac{1}{RC_M}\Theta_c(t) + \frac{UA}{C_M}\Big\{q_c(t) - \Theta_0(t)\Big\} \tag{2.2}$$

R and C_M in Equation (2.2) are the same parameters as Equation (2.1). With respect to the parameter A, A is represented by the diameter of the tube d and the effective contact length with an aqueous solution L. Therefore, A is expressed as follows:

$$A = \pi\left(\frac{d}{2}\right)^2 L \tag{2.3}$$

Thus,A is a known constant.

Now, let the control variable U is represented to $U(v,t)$, where is the overall heat transfer coefficient. $v[m/s]$ is the speed of the cooling water. This system is described by the bilinear differential equation, where is nonlinear coupling between the control variable and state variable.

For simplicity, let $\Theta_c(t)$ and $q_c(t)$ to a constant value respectively as follows:

$$\Theta_c \equiv \Theta(t), \ \ q_c \equiv q_c(t) \tag{2.4}$$

Then, Equation (2.2) is as follows:

$$\frac{d\Theta_0(t)}{dt} = \frac{U(v,t)A}{C_M}\Big\{q_c - \Theta_0(t)\Big\} + \frac{1}{RC_M}\Big\{\Theta_c - \Theta_0(t)\Big\} \tag{2.5}$$

We obtain as follows by replacing Equation (2.5).

$$\frac{d\Theta_0(t)}{dt} = w(t)g(\Theta_0(t)) + f(\Theta_0(t)) \tag{2.6}$$

where

$$w(t) \equiv \frac{A}{C_M}U(v,t) \tag{2.7}$$

$$g(\Theta_0(t)) \equiv q_c - \Theta_0(t)$$
$$f(\Theta_0(t)) \equiv \frac{1}{RC_M}\Big\{\Theta_c - \Theta_0(t)\Big\} \tag{2.8}$$

We can obtain as follows by linearizing Equation (2.6), which is referred in Appendix A.

$$\frac{d\xi(t)}{dt} = -a\xi(t) + bu(t) \tag{2.9}$$

where

$$a \equiv +\frac{1}{RC_M}, \ \ \xi(t) \equiv \Theta_0(t) - \Theta_c(t)$$
$$w(t) \equiv \frac{1}{q_c - \Theta_0}\left[\frac{1}{RC_M}(\Theta_c - \Theta_0) + u(t)\right] \times \frac{A}{C_M} \tag{2.10}$$

As a result, Equation (2.9) represents the linear differential equation.

3   One-dimensional outlet temperature predictive control system

The exact linear ordinary differential equation model is derived as follows:

$$\frac{d\theta(t)}{dt} = a\theta(t) + bg(t) \tag{3.1}$$

where, the following equation holds (see Appendix A).

$$a = \left\{ \frac{1}{RC_M} + \frac{k_1}{RC_M^2} \right\} \tag{3.2}$$

$$b = \frac{A}{C_M} \tag{3.3}$$

$$\theta(t) = k_0 u_c(t) - k_1 u(t, x_L) \tag{3.4}$$

At this time, from the observer theory known as state estimation in control engineering, we obtain as follows:

$$\frac{dw}{dt} = Fw + Gy + Tbg, \quad y = C(u) \tag{3.5}$$

When we configure the observer from the observer theory, we obtain as follows:

$$TA - FT = GC$$
$$w(t) = Tu + \exp(Ft)\{w(0) - Tu(0)\} \tag{3.6}$$

Consequently, the estimation value is derived as follows[9]:

$$w(t) = T^{-1}\theta(t) \tag{3.7}$$

Then, we unify the system variables as follows in this paper.

$$\frac{d\theta(t)}{dt} = a\theta(t) + bu(t) + d_n\xi(t)$$
$$y(t) = C\theta(t) \tag{3.8}$$

3.1 Optimal servo system

We construct a regulator system by introducing the estimator of the equivalent input disturbance[9].
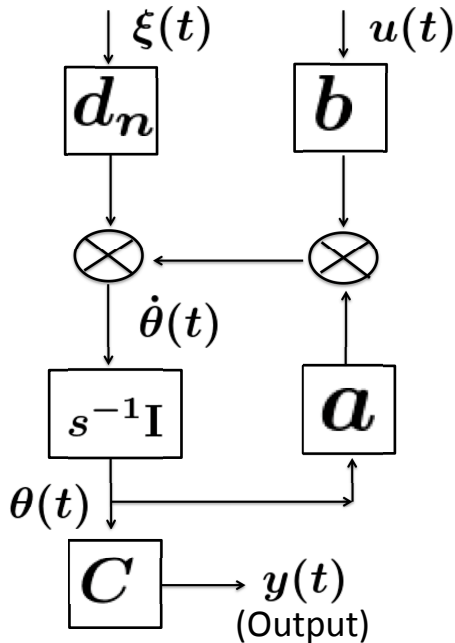

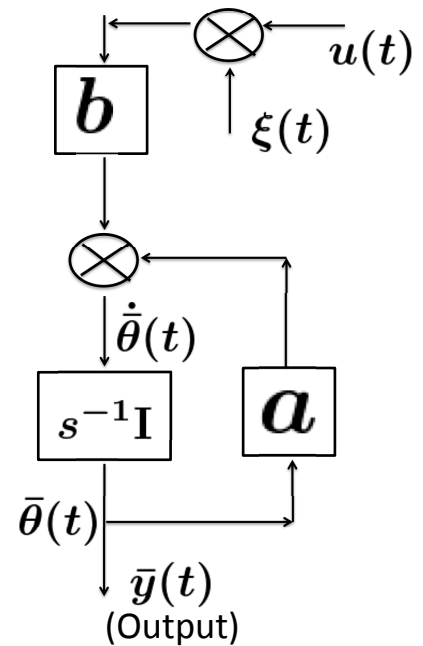Fig. 3: Exact linear ordinary differential equation model with disturbance

Fig. 4: Equivalent input disturbance system model

From Figure (3) and Figure (4), we obtain as follows:

$$\frac{d\bar{\theta}(t)}{dt} = a\bar{\theta}(t) + b\{u(t) + \bar{\xi}(t)\} \tag{3.9}$$

$$\bar{y}(t) = C\bar{\theta}(t) \tag{3.10}$$

where Equations (3.9) - (3.10) satisfy as follows.

$$\bar{\theta}(t) = \hat{\theta}(t) - \theta_R(t) \tag{3.11}$$

$$\frac{d\theta_R}{dt} = a\theta_R(t) + b\Delta\xi(t) \tag{3.12}$$

We obtain as follows after substituting Equation (3.11) to Equation (3.9).

$$\frac{d\hat{\theta}(t)}{dt} = a\hat{\theta}(t) + bu(t) + b\left\{\bar{\xi}(t) + \left[\frac{d\theta_R(t)}{dt} - a\theta_R(t)\right]\right\} \tag{3.13}$$

Further, we obtain as follows after substituting Equation (3.12) to Equation (3.13).

$$\begin{aligned}\frac{d\hat{\theta}(t)}{dt} &= a\hat{\theta}(t) + bu(t) + \left\{b\bar{\xi}(t) + \left[b\Delta\xi(t)\right]\right\} \\ &= a\hat{\theta}(t) + b\left\{u(t) + (\bar{\xi}(t) + \Delta\xi(t))\right\} \\ &= a\hat{\theta}(t) + b(u(t) + \hat{\xi}(t))\end{aligned} \tag{3.14}$$

where we replaced as follows:

$$\hat{\xi}(t) = \bar{\xi}(t) + \Delta\xi(t) \tag{3.15}$$

From Equations (3.14) - (3.15), the difference between the estimated data $\hat{\theta}(t)$ obtained by Observer theory and the actual data obtained from the real work is equal to the difference of equivalent input disturbance.

We get as follows after putting the observer gain to L.

$$\frac{d\hat{\theta}(t)}{dt} = a\hat{\theta}(t) + bu^*(t) + LC[\theta(t) - \hat{\theta}(t)] \tag{3.16}$$

where $u^*(t)$ represents a optimal control input and satisfies as follows:

$$u^*(t) = -K_p\hat{\theta(t)} + K_R\theta_R(t) \tag{3.17}$$

Then, we obtain as follows by using Equation (3.14).

$$a\hat{\theta}(t) + b(u(t) + \hat{\xi}(t)) = a\hat{\theta}(t) + bu^*(t) + LC[\theta(t) - \hat{\theta}(t)] \tag{3.18}$$

Further, from Equation (3.18), we obtain as follows:

$$b\hat{\xi}(t)) = b[u^*(t) - u(t)] + LC[\theta(t) - \hat{\theta}(t)] \tag{3.19}$$

Accordingly, from Equation (3.19), we obtain as follows:

$$\hat{\xi}(t)) = \frac{LC}{b}[\theta(t) - \hat{\theta}(t)] + [u^*(t) - u(t)] \tag{3.20}$$

where we call θ_R as an internal model, θ_R is derived as follows:

$$\frac{d\theta_R}{dt} = a_R\theta_R(t) + b_R[r(t) - y(t)]$$
$$y(t) = C\theta(t) \tag{3.21}$$

where $r(t)$ is a control target input.

Figure 5 shows the internal model, and K_R denotes a optimal gain of open loop system for the internal model and K_P denotes a optimal gain for $u(t)$. In Figure 6, $\tilde{\xi}(t)$ denoted a final estimated value of the equivalent input disturbance. $F(s)$ denotes a lowpass filter.
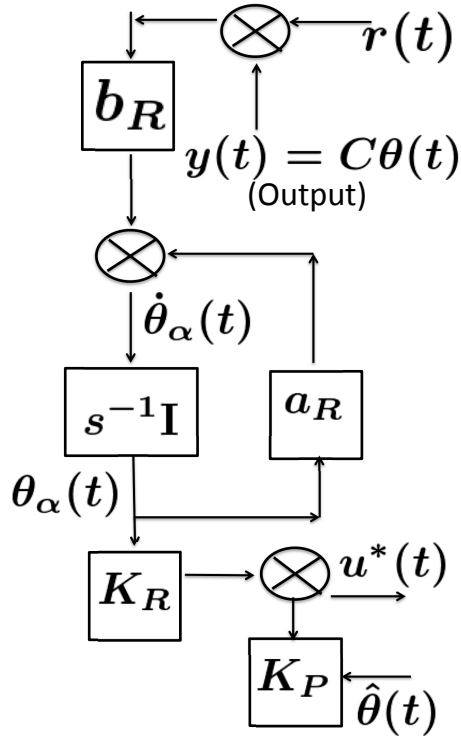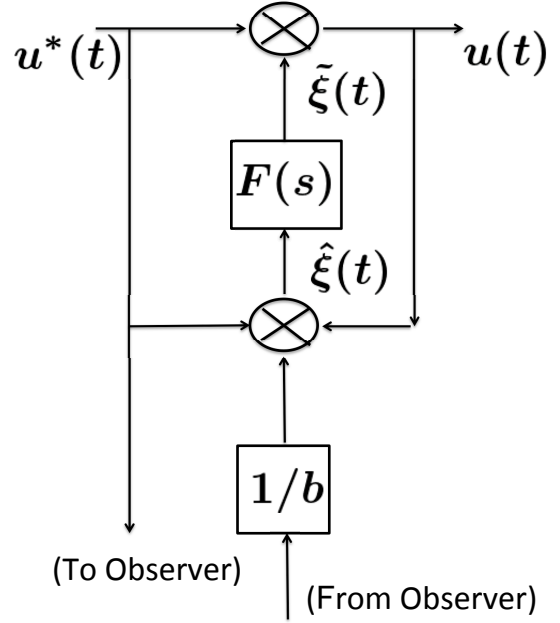
Fig. 5: Internal model

Fig. 6: Estimator for disturbance

Definition 3.1

$$\tilde{D}(s) \equiv \mathscr{L}_s[\tilde{\xi}(t)] \tag{3.22}$$

$$\hat{D}(s) \equiv \mathscr{L}_s[\hat{\xi}(t)] \tag{3.23}$$

where, $\mathscr{L}_s$ denotes a Laplace transformation.

From Figure 6, we obtain as follows:

$$\tilde{D}(s) = F_\xi(s)\hat{D}(s) \tag{3.24}$$

$$u(t) = u^*(t) - \hat{\xi}(t) \tag{3.25}$$

With respect to Equation (3.25), the control input $u(t)$ substitute the optimal control function $u^*(t)$ to the estimation value of equivalent disturbance $\hat{\xi}(t)$. Also, With respect to Equation (3.26), the lowpass filter $F(s)$ is derived as follows:

$$|F_\xi(j\omega)| \simeq 1, \ \omega \in \Omega_r \ (\Omega_r = \{\omega \leq \omega_r\}) \tag{3.26}$$

The transfer function $G(s)$ from $\tilde{\xi}(t)$ to $\hat{\xi}(t)$ is derived as follows:

$$G(s) = \frac{b(s-a)(s-(a-LC))^{-1}b}{b^2} \tag{3.27}$$

On the other hand, $G(s)$ is restricted as follows:

$$|G(s) \cdot F_\xi(s)|_\infty < 1 \tag{3.28}$$

Namely, the observer gain L should be designed to satisfy the following equation:

$$G(j\omega) < \frac{1}{|F_\xi(j\omega)|}, \ \omega \in [0,\infty] \tag{3.29}$$

3.2 Determination of the observer gain

We define the following equation to determine of the observer gain.

Definition 3.2

$$\frac{d\theta^L(t)}{dt} = a\theta^L(t) + Cu^L(t) \tag{3.30}$$

$$y_L = C\theta^L(t) \tag{3.31}$$

Definition 3.3 *From the feedback control law for the evaluation function to minimize observer gain L_ρ is determined by the following equation.*

$$J_L = \int_0^\infty [\rho Q_L(\theta^L(t))^2 + R_L(u^L(t))^2]dt \tag{3.32}$$

$$u_{opt}^L(t) = -L_\rho \theta^L(t) \tag{3.33}$$

$$L_\rho \equiv R_\rho^{-1} CP(t) \tag{3.34}$$

$$-\frac{dP}{dt} = 2aP + R_L C^2 P^2 - (\rho Q_L)^2 \tag{3.35}$$

where, we are going to determine L_ρ including $\rho \geq 0$.

With respect to Equation (3.31), the parameters (a,b,c) satisfy the following equation under a minimum phase system.

$$\lim_{\rho \to \infty} \left[s - (a - L_\rho C) \right]^{-1} b = 0 \tag{3.36}$$

Therefore, the optimal observer gain L_ρ was found to exist.

Then, we replace Equation (3.36) to φ as follows:

$$\varphi(s) = \left[s - (a - L_\rho C) \right]^{-1} b \tag{3.37}$$

As $\varphi(s)$ constitutes a part of $G(s)$ in Equation (3.27), $|G(j\omega)|$ can be small value under a large value ρ sufficiently, that is, L_ρ and $F(s)$ that satisfy Equation (3.28) become possible to design.

From Equations (3.8), (3.21) and (3.25), we describe the vector differential equation as follows:

$$\frac{d}{dt}\begin{bmatrix} \theta(t) \\ \theta_R(t) \end{bmatrix} = \begin{bmatrix} a & 0 \\ -c & a_R \end{bmatrix}\begin{bmatrix} \theta(t) \\ \theta_R(t) \end{bmatrix} + \begin{bmatrix} b \\ b_R \end{bmatrix}\begin{bmatrix} u^*(t) \\ r(t) \end{bmatrix} \tag{3.38}$$

where, $u^*(t) = u(t) + \hat{\xi}(t)$

We rewrite Equation (3.38) as follows:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}_x \mathbf{x}(t) + \mathbf{B}_x \mathbf{u}_f(t) \tag{3.39}$$

where,

$$\mathbf{A}_x = \begin{bmatrix} a & 0 \\ -c & a_R \end{bmatrix}, \qquad \mathbf{B}_x^T = \begin{bmatrix} b & b_R \end{bmatrix} \tag{3.40}$$

$$\mathbf{x}(t) = \begin{bmatrix} \theta(t) & \theta_R(t) \end{bmatrix}^T, \qquad \mathbf{u}_f(t) = \begin{bmatrix} u^*(t) & r(t) \end{bmatrix} \tag{3.41}$$

The evaluation function J_K for the model of Equation (3.39) is defined by the following equation to calculate the optimal feedback gain.

Definition 3.4

$$J_K = \int_0^T [\mathbf{x}^T Q_K \mathbf{x} + \mathbf{u}_f^T R_K \mathbf{u}_f]dt \tag{3.42}$$

The optimal control law is derived as follows:

$$\mathbf{u}_f^{opt}(t) = -\mathbf{K}(t)\mathbf{x}(t) \tag{3.43}$$

$$\mathbf{K} = [K_P, K_R] \tag{3.44}$$

where,

$$K(t) = \mathbf{R}_K^{-1}\mathbf{B}_x\mathbf{P}(t) \tag{3.45}$$

$$-\frac{dP}{dt} = 2\mathbf{A}_x\mathbf{P} + \mathbf{R}_K\mathbf{B}_x^2\mathbf{P}^2 - \mathbf{Q}_K^2, \qquad \mathbf{P}(T) = 0 \tag{3.46}$$

At this time, we take a low-pass filter $F(s)$ in the following equation[9].

$$F_\xi(s) = \frac{1}{1 + T_s} \tag{3.47}$$

From the above description, the design of optimal adaptive control becomes possible to improve a disturbance rejection performance.

4   Numerical example

We consider a mathematical model as follows:

$$\frac{d}{dt}\begin{bmatrix} \theta(t) \\ \theta_R(t) \end{bmatrix} = \begin{bmatrix} 0.8 & 0 \\ -1 & 0.6 \end{bmatrix}\begin{bmatrix} \theta(t) \\ \theta_R(t) \end{bmatrix} + \begin{bmatrix} 1.5 & 1 \end{bmatrix}\begin{bmatrix} u(t) \\ r(t) \end{bmatrix} \tag{4.1}$$

$$\mathbf{Q} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \qquad R = \frac{1}{2} \tag{4.2}$$

Then, the mathematical model Equation (3.30) is derived as follows:

$$\frac{d\theta_L(t)}{dt} = 0.8\theta_L(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u_L(t) \tag{4.3}$$

$$Q_L = 1, \qquad R_L = \frac{1}{4} \tag{4.4}$$

The Riccati equation of stationary system derived as Equations (??), (??) and (??) is as follows:

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} 0.8 & 0 \\ -1 & 0.6 \end{bmatrix} + \begin{bmatrix} 0.8 & -1 \\ 0 & 0.6 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$
$$-\left(\frac{1}{2}\right)^{-1}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} 1.5 & 1 \end{bmatrix}\begin{bmatrix} 1.5 \\ 1 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$
$$+\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = 0 \tag{4.5}$$

Therefore, the optimal gain is derived as follows:

$$\begin{bmatrix} K_P \\ K_R \end{bmatrix} = -\left[\frac{1}{2}\right]^{-1}\begin{bmatrix} 1.5 & 1 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{4.6}$$

As a results, the optimal control function $u(t)$ is derived as follows:

$$u(t) = \begin{bmatrix} K_P & K_R \end{bmatrix}\begin{bmatrix} \theta_L(t) \\ \theta_R(t) \end{bmatrix} \tag{4.7}$$

Consequently, the observer system P_1 is derived as follows:

$$2 \times 0.8P_1 - \left(\frac{1}{4}\right)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}^2 P_1^2 + f^2 = 0 \tag{4.8}$$

Moreover, the observer gain L_f is derived as follows:

$$L_f = \left(\frac{1}{4}\right)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} P_1 \tag{4.9}$$

Finally, the optimal input $u_L(t)$ is derived as follows:

$$u_L = -L_f \theta_L(t) \tag{4.10}$$

P can be calculated From Equation (4.5) as follows:

$$\mathbf{P} = \begin{bmatrix} 1.27 & 0.4 \\ 0.79 & 1.11 \end{bmatrix} \tag{4.11}$$

where, as $\mathbf{P} > 0$, Therefore, the optimal gain is derived from Equation (4.6) as follows:

$$\begin{bmatrix} K_P \\ K_R \end{bmatrix} = \begin{bmatrix} -10.78 \\ -6.84 \end{bmatrix} \tag{4.12}$$

Thus, the optimal control is derived as follows:

$$\begin{bmatrix} u(t) \\ u_R(t) \end{bmatrix} = \begin{bmatrix} -10.78\theta(t) \\ -6.84\theta_R(t) \end{bmatrix} \tag{4.13}$$

The optimal observer gain is derived from Equation (4.9) as follows:

$$L_f = \left(0.8 + \frac{\sqrt{256 + 16f^2}}{2}\right) \tag{4.14}$$

5 Results

When we can transform into a linear model using an exact linearization theory, we clarified that we can easily expand a linear model by using that method. We will report these on a next time that the selection of the low pass filter and the design method by using an estimation method other than the observer theory discussed in this study. Moreover, the discussion, including the extension to the distributed parameter system in the exact sense will be discussed over next time.

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A   Appendix Exact linearization

In Exact Linearization Method, we take advantage of Lie derivative.
In such a model of this study, we put

$$g(\Theta_0) = q_c - \Theta_0 \tag{A.1}$$

$$f(\Theta_0) = \frac{1}{RC_M}(\Theta_c - \Theta_0) \tag{A.2}$$

$$\frac{d\Theta_0}{dt} = f(\Theta_0) + g(\Theta_0)w(t) \tag{A.3}$$

Therefore, it is to satisfy the theorem of Exact Linearization Method[8].
Consequently, we obtain as follows:

$$ad_f^0 g\phi(\Theta_0) = (q_c - \Theta_0) \tag{A.4}$$

$$ad_f^1 g\phi(\Theta_0) = [f \cdot g](\Theta_0) = \frac{\partial g}{\partial \Theta_0}f(\Theta_c) - \frac{\partial f}{(\partial \Theta_0)}g(\Theta_0)$$

$$= -\frac{1}{RC_M}(\Theta_c - \Theta_0) + (T_c - \Theta_0)\frac{1}{RC_M}$$

$$= \frac{1}{RC_M}(2\Theta_0 + q_c - \Theta_c)$$

$$ad_f^2 g(\Theta_0) = [f \cdot ad_f^1 g]f(\Theta_0)$$

$$= [\frac{\partial}{\partial \Theta_0}ad_f^1 g]f(\Theta_0) - \frac{\partial f}{\partial \Theta_0}ad_f^1 g(\Theta_0)$$

$$= \frac{2}{RC_M} \times \frac{1}{RC_M}(\Theta_c - \Theta_0) + \frac{1}{RC_M}(2\Theta_0 - q_c - \Theta_c) \times \frac{1}{RC_M}$$

$$= \left(\frac{1}{RC_M}\right)^2 \left\{2(\Theta_c - \Theta_0) + 2\Theta_0 - q_c - \Theta_c\right\}$$

$$= \left(\frac{1}{RC_M}\right)^2 (\Theta_c - q_c) \tag{A.5}$$

$$[ad_f^0 g, ad_f^0 g](\Theta_0) = \left[\frac{\partial}{\partial \Theta_0} ad_f^0 g(\Theta_0) \right] ad_f^0 g(\Theta_0) - \frac{\partial [ad_f^0 g(\Theta_0)]}{\partial \Theta_0} ad_f^0 g(\Theta_0)$$

$$= -(q_c - \Theta_0) + (q_c - \Theta_0) = 0$$

$$[ad_f^1 g, ad_f^1 g](\Theta_0) = \left[\frac{\partial}{\partial \Theta_0} ad_f^1 g(\Theta_0) \right] ad_f^1 g(\Theta_0) - \frac{\partial [ad_f^1 g(\Theta_0)]}{\partial \Theta_0} ad_f^1 g(\Theta_0)$$

$$= \frac{1}{RC_M} \times 2 \frac{1}{RC_M} (2\Theta_0 - q_c - \Theta_c) - \frac{2}{RC_M} \times \frac{1}{RC_M} (2\Theta_0 - q_c - \Theta_c) = 0$$

$$[ad_f^1 g, ad_f^1 g](\Theta_0) = \left[\frac{\partial}{\partial \Theta_0} ad_f^0 g(\Theta_0) \right] ad_f^1 g(\Theta_0) - \frac{\partial [ad_f^1 g(\Theta_0)]}{\partial \Theta_0} ad_f^0 g(\Theta_0)$$

$$= span[ad_f^0 g, ad_f^1 g](\Theta_0) \tag{A.6}$$

As a results we obtain as follows:

$$\mathscr{L} ad_f^0 g \phi(\Theta_0) = \left(\frac{\partial \phi}{\partial \Theta_0} ad_f^0 g(\Theta_0) \right) = \frac{\partial \phi}{\partial \Theta_0} (q_c - \Theta_0)$$

$$\mathscr{L} ad_f^1 g \phi(\Theta_0) = \left(\frac{\partial \phi}{\partial \Theta_0} ad_f^1 g(\Theta_0) \right) = \frac{\partial \phi}{\partial \Theta_0} \times \frac{1}{RC_M} (2\Theta_0 - q_c - \Theta_c) \tag{A.7}$$

Therefore, let $\phi(\Theta_0) = \Theta_0 - \Theta_c$ and $w(t)$ is derived as follows:

$$w(t) = \left[\frac{f(\Theta_0)}{q_c - \Theta_0} + \frac{u(t)}{q_c - \Theta_0} \right] \times \frac{A}{RC_M}$$

$$= \frac{1}{q_c - \Theta_0} \left\{ \frac{1}{C_M} \left(\Theta_c - \Theta_0 \right) + u(t) \right\} \times \frac{A}{C_M} \tag{A.8}$$

where let $\xi \equiv \phi(\Theta_0) \equiv \Theta_0 - \Theta_c$ and we can rewrite to

$$\frac{d\xi}{dt} = \frac{d\xi}{d\Theta_0} \cdot \frac{d\Theta_0}{dt}$$

$$= \frac{1}{RC_M} (\Theta_c - \Theta_0) + (q_c - \Theta_0) w(t)$$

$$= + \left\{ \frac{1}{RC_M} (\Theta_c - \Theta_0) + (q_c - \Theta_0) \left[\frac{A}{C_M(q_c - \Theta_0)} \left\{ \frac{1}{RC_M} (\Theta_c - \Theta_0) + u(t) \right\} \right] \right.$$

$$= - \left\{ \frac{1}{RC_M} + \frac{1}{RC_M^2} \right\} (\Theta_0 - \Theta_c) + \frac{A}{C_M} u(t) \tag{A.9}$$

where,

$$a = \left[\frac{1}{RC_M} + \frac{A}{RC_M^2} \right], \quad b = \frac{A}{RC_M^2} \tag{A.10}$$

That is, the model of Exact Linearization Method is expressed by

$$\frac{d\xi}{dt} = -a\xi(t) + bu(t). \tag{A.11}$$